Letters to the Editor

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ON THE NON-COHERENT FORMATION OF ABSORPTION LINES IN STELLAR ATMOSPHERES

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The equation of transfer for partly coherent and partly non-coherent formation of absorption lines in stellar atmospheres can be written as

$$\mu \frac{dI_\nu(t,\mu)}{dt} = (1+\eta_\nu)I_\nu(t,\mu)+(1-\epsilon)\cdot a\cdot \eta_\nu J_\nu(t)+(1+\epsilon\eta_\nu)B_\nu(t)+(1-\epsilon)b\cdot\eta_\nu J(t), \quad \ldots \quad (1)$$

where I_ν (t, μ) is the specific intensity of radiation of frequency ν in the direction $\theta = \cos^{-1}\mu$ and at the optical depth t measured in the continuous absorption coefficient k_ν, k_ν being constant over the width of the line considered. $t = 0$ corresponds to the outer surface of the atmosphere supposed to be stratefied in plane parallel layers. $J_\nu(t)$ is the average intensity.

ϵ = fraction of atoms excited by radiation of frequency ν and prevented from contributing to the coherent re-emission by superelastic collisions. ϵ is however much less than 1.

$$\eta_\nu = \frac{l_\nu}{k_\nu} = \frac{\text{line absorption co-efficient}}{\text{continuous absorption co-efficient}}, \quad (2)$$

is assumed to be independent of the optical depth.

$$a+b = 1, \; a \text{ and } b \text{ are both positive.} \quad \ldots \quad (3)$$

The second term on the right hand side of (1) gives the coherent part of emission while the last term in the equation (1) accounts for the non-coherent part of emission. $J(t)$ will be called pseudo average intensity and will be regarded to be insensitive to the variation of frequency over the absorption line considered. The boundary conditions are

$$I_\nu(0, -\mu') = 0, \; 0 < \mu' \leqslant 1, \quad \ldots \quad (4)$$

$$I_\nu(t,\mu)\cdot\exp(-t/\mu\lambda)\to 0 \text{ as } t\to\infty, \quad \lambda = 1/(1+\eta_\nu), \qquad \dots (5)$$

$$J(t)\to B_\nu(t) \text{ as } t\to\infty. \qquad \dots (6)$$

where $B_\nu(t)$ is the Planck's function assumed to be constant over the line. We now put

$$\lambda = 1/(1+\eta_\nu),\ \gamma = 1+c\eta_\nu,\ \sigma = (1-\epsilon)\cdot\eta_\nu, \qquad \dots (7)$$

$$C(\mu) = \left\{\frac{\partial I_\nu(0,\mu)}{\partial\sigma}\right\}_{\sigma\to}, \quad E_n(t) = \int_1^\infty \exp(-xt)dx/x^n. \qquad (8)$$

$$D_{r+1}(t) = \frac{t^r}{1\cdot r!} - \frac{t^{r-1}}{2\cdot(r-1)!} + \dots \frac{(-1)^r}{(r+1)}, \qquad (9)$$

$$\phi_{r+1}(t) = D_{r+1}(t)+(-1)^{r+1}E_{r+2}(t), \qquad (10)$$

$\omega' = \sigma\cdot\lambda,\ \omega = a\cdot\omega'$, both ω, ω' lie between 0 and 1.

$$T(z) = 1-\omega\tfrac{1}{2}z\ln\frac{z+1}{z-1}, \quad L_1(z) = 1-\omega'\cdot\lambda\cdot z\ln\frac{z+1/\lambda}{z}. \qquad (12)$$

$H(z)$ is the unique solution of the integral equation

$$T(z)\,H(z) = \omega\tfrac{1}{2}\int_0^1 \frac{xH(x)dx}{x-z} + \sqrt{(1-\omega)}, \qquad (13)$$

and

$$H(-z)\to 1+(\omega-R_0)\cdot z\cdot\ln z+0(z) \text{ as } z\to 0, \qquad (14)$$

We further put

$$\omega_0 = \omega'(1+\lambda)-R_0, \qquad (15)$$

$$L_2(z) = 1-\omega_0 z\ln\frac{z+1}{z}, \qquad (16)$$

$$U(z) = H(-z)L_1(z) \quad L_2(z) = u_0+U_-(z)-U_+(z), \qquad (17)$$

where $u_0 = \underset{z\to\infty}{Lt}U(z)$ and $U_-(z)$ is a function regular outside a loop enclosing the branch cut $(0, -1/\lambda)$ on the left of the imaginary axis, while $U_+(z)$ is a function regular outside a loop enclosing the cut (0, 1) and the pole K (real) which is the positive real root of $T(z)$ outside the cut (0, 1).

$$B_\nu(t) = \sum_0^N b_r t^r, \quad g^0(z) = -\tfrac{1}{2}\sum_0^N b_r r!\,\lambda^r z^r, \qquad \dots (18)$$

$$Q^0(z) = \sum_0^N Q_r^0 z^r, \quad Q^0_r = \tfrac{1}{2}\sum_{s=0}^N \frac{b_{r+s}(r+s)!}{s+1} \qquad \dots (19)$$

$$R(z) = \sum_0^N \frac{r+1-\epsilon}{1-\epsilon}\, b_r\cdot r!z^r. \qquad \dots (20)$$

$$f^0(z) = \sum^N \{b\omega'(Q^0_r+C^0_r)+\lambda\gamma b_r r!\}\lambda^r z^r, \qquad \dots (21)$$

where $C^0(z) = \Sigma\, c_r^0 z^r$, c_r^0's are determined from the boundary condition (6), (22)

δ_0 = wing-damping constant of a broadened line, (23)

δ = damping constant of the profile of the absorption co-efficient, (24)

$r_0(\mu)$ = central residual intensity of the line, (25)

$$I^c_\nu(0, \mu) = \sum^N b_r \cdot r! \cdot \mu^r = \text{intensity in the continuum}, \tag{26}$$

$$\beta = \frac{\delta_0^2}{(1-\epsilon)\delta^2\eta_\nu}, \quad G(z) = \tfrac{1}{2}\omega' \int_0^1 \frac{xI_\nu(0, x)dx}{x-z} \tag{27}$$

The solution of the integro-differential equation (1) is obtained with the help of Laplace- transformation in combination with the principle of analytic continuation. In obtaining our solution it has been possible to avoid the normal practice of assuming an approximate form of $J(t)$. We have been able to find an explicit, closed expression for $J(t)$ by a new approach based on the expansion of I_ν $(0, \mu)$ in powers of σ (justification for this expansion has been advanced).

If $C(z)$ in (8) is assumed to be a polynomial in z(an assumption analytically unjustifiable) we can find an expression depending on the coefficients b_r's and a_r's provided $r_0(z)$ can be determined by $r_0(z) = \sum\limits_0 a_r z^r / I_\nu^c(0, z)$. The solution of (1) is then obtained on this basis.

However an analytically correct expression for $C(z)$ has been found and is given by

$$C(z) = Q^0(z) + \{\sum_0^N b_r r!\, z^r\}\, \tfrac{1}{2}\, z \ln \frac{z+1}{z} - R(z) + b.C^0(z). \quad \dots \tag{28}$$

and then the pseudo average intensity $J(t)$ is obtained as

$$J(t) = \sum_0^N Q_r^0 t^r/r! + \sum_0^N C_r^0 t^r/r! + \tfrac{1}{2}\sum_0^N b_r r!\phi_{r+1}(t). \quad \dots \tag{29}$$

After getting an expression for $J(t)$ as given by (29) it is possible to solve the equation (1) and the emergent intensity is then obtained and given by

$$I_\nu(0, z) = H(z)\, A^0(z) + bH(z) g^0(z)\{U_-(z) + L_2(z)\}, \quad \dots \tag{30}$$

where

$$A^0(z) = \underset{z\to\infty}{\text{Lt.}} \{bg^0(z)[u_0 - U_+(z)] + H(-z)[aG(z) + f^0(z) - bg^0(z)]\} \quad \dots \tag{31}$$

From the solution (30) we can get the emergent intensity for coherent scattering simply by putting $b = 0$ in it (as we should). The solution based on the polynomial representation of $C(z)$ is defective in this respect. The solution for completely non-coherent scattering is deduced by putting $a = 0$ in (30). The expression (29) giving $J(t)$ is interesting.

REFERENCE

W. Busbridge, M. N., 1953, **113**, 52,.